Evaluation of the Security of My-Ellty ECMR

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Chapter 1

Introduction

The goal of this document is to evaluate the security of the digital signature schemes My-Ellty ECMR 160-h, My-Ellty 192-h, und My-Ellty OEF-h.

The security of the My-Ellty schemes depends on the intractability of the discrete logarithm problem in point groups of elliptic curves over finite fields (ECDL) and on the security of the used hash function and pseudorandom number generator. In order to evaluate the security of the My-Ellty schemes, it is, therefore, necessary to evaluate the difficulty of ECDL and the security of the used hash function and pseudorandom number generator. But this is not sufficient. Even if the underlying number theoretic problem is hard and the hash function and random number generator are secure, the My-Ellty schemes may still be insecure. A famous example for such a situation is the discovery of the possibility of an attack against the RSA encryption standard PKCS #1 (see [2]). In this standard, an insecure padding scheme was used, which compromised the security of the whole scheme.

Because of this situation, it is desirable to find a proof for the security of the My-Ellty schemes. Unfortunately, no provably hard computational problem in number theory is known, which could serve as the basis of a secure signature scheme. Also, no provably secure cryptographic hash function and pseudorandom number generator are known. Therefore, given current knowledge, there are no provably secure digital signature schemes. But it is possible to say more about the security of a digital signature scheme than just arguing that the underlying computational problems are intractable. Modern security proofs for digital signature schemes reduce their security to the difficulty of basic computational problems in mathematics. This means that the difficulty of those basic problems is not only necessary but also sufficient

for the security of the digital signature scheme which relies on their security.

The questions that I answer in this report are the following. Which are the basic computational problems, on which the security of the My-Ellty schemes is based, and to what extent can this security be reduced to the intractability of those problems? How difficult are those basic mathematical problems and how difficult are the instances which arise from the specific applications in the My-Ellty schemes?

This report is organized as follows. Chapter 2 gives an overview over the My-Ellty schemes. Chapter 3 describes the models and techniques that are used in the security proofs of digital signature schemes and explains to what extent the security of the My-Ellty schemes can be proved secure in those models. The security of the signature schemes under review relies on the intractability of computing discrete logarithms in elliptic curve point groups, finding collisions of hash functions and guessing the output of pseudorandom number generators. Chapter 4 describes the current knowledge concerning the intractability of those basic problems. Chapter 5 describes the specific instances of the basic computational problems on which the security of the My-Ellty schemes rely and their difficulty. I conclude this report by summarizing the security of the My-Ellty schemes in Chapter 6.

Chapter 2

Description of the My-Ellty schemes

2.1 The setup

I describe the structure of a digital signature scheme. A digital signature scheme has three parts.

Key generation The signer generates a private key and the corresponding public key. He keeps the private key secret and publishes the public key. The authenticity of the public key is certified by a certification authority (CA). The certification authority guarantees with its signature that the verifiers obtain the valid public keys of the signers. It is also possible that the certification authority generates the key pair and gives the secret key to the user.

Signature generation In this step the signer produces the digital signature of a document d. To generate its digital signature, the signer uses his private key.

Signature verification The verifier uses the public key of the signer to verify the digital signature. The signature is convincing if only the signer, knowing his private key, is able to produce that valid signature. The existence of that valid signature then implies that the signer must have produced it, thereby agreeing to the content of the document.

In a digital signature with *message recovery* the verifier obtains the signed document from the verification procedure. If the digital signature system does not allow message recovery, then the verifier also needs the document d as an input to verify the signature. The schemes ACE Sign, ECDSA, ESIGN do not have message recovery. The My-Ellty systems have partial message recovery, i.e., in the verification process, the verifier obtains part of the signed message. The remaining part of the message is also sent to the verifier.

2.2 My-Ellty

I give a summary of My-Ellty. Technical details can be found in the submission.

For each of the signature schemes My-Ellty ECMR OEF-h, My-Ellty ECMR 192-h, and My-Ellty ECMR 160-h a finite field, an elliptic curve over that field, and a point on that curve of prime order is specified. The specification of those data can be found in the proposal.

For the description of the algorithms I let F be one of those fields with the corresponding curve E, point G, and point order l.

2.2.1 Key generation

1. An integer $v \in \{1, \ldots l-1\}$ is chosen.
2. The point $Y = vG$ is computed.

Presumably, v is a random number. This is missing in the description. The secret key is v. The public key is Y.

2.2.2 Signature generation

The document to be signed is d, a bit string of arbitrary length. In the description of the signature generation, we use the integer u which is 96 for My-Ellty ECMR 192-h and 80 for My-Ellty ECMR OEF-h and My-Ellty ECMR 160-h. In the signature generation process, a hash function $h : \{0,1\}^* \to \{0,1\}^u$ is used which is constructed using the SHA-1 hash function(see [16]). The signer uses the private key from the previous section and executes the following steps.

1. Let d_1 be the string consisting of the first u bits of the document d to be signed, let d_2 be the remaining string, and let $h = h(d)$ be the hash value of d. Set $m = d_1||h$. Then m is a length $2u$ bit string.
2. Generate a random $k \in \{1, \ldots, l-1\}$.
3. Compute the point $(x, y) = kG$ on the elliptic curve E.
4. Compute $r = m \oplus x$.
5. Compute $r' = r \bmod l$. If $r' = 0$ return to step 2.
6. Compute $s = (r'k - r' - 1)(v+1)^{-1} \bmod l$. If $s = 0$ return to step 2.
7. Output the signature (r, s) together with the string d_2.

2.2.3 Signature verification

I explain the verification of the signature from the previous section which uses the public key from Section 2.2.1. The verifier executes the following steps.

1. Compute $r' = r \bmod l$.
2. Verify that $r' \neq 0$ and $0 < s < l$. If this is not satisfied, reject.
3. Compute the point $(x, y) = (1+r'+s)(r')^{-1}G + s(r')^{-1}Y$ on the elliptic curve.
4. Compute $m = r \oplus x$.
5. Decompose m into two strings of length u. Call the first string d_1 and the second string h'.
6. Compute the message $d = d_1||d_2$.
7. Determine the hash value $h = h(d)$.
8. If $h = h'$ accept the signature. Otherwise, reject it.

Chapter 3

Security proofs

In this chapter I explain what a security proof for a digital signature system is and which security proofs for the My-Ellty schemes are given in the self evaluation.

3.1 Security of signature schemes

3.1.1 Security proofs are reductions

The security of all known digital signature schemes depends on the intractability of certain computational problems in mathematics, specifically in number theory. Examples are the integer factoring problem and the discrete logarithm problem in an appropriate group. However, no provably hard computational problems are known which can serve as the security basis of a digital signature scheme. Therefore, no rigorous security proofs for signature schemes are known and there is little hope that such proofs will be found in the future.

Todays security proofs are reductions. The goal of such a reduction is to show that the ability of an attacker to mount a successful attack on a signature scheme implies his ability of solving a basic computational problem in mathematics. This is supposed to increase the trust in the security of a digital signature system. The idea of this approach is the following.

When analyzing the security of a signature scheme it is hard to predict which attacks are possible since the system may be very complex and may depend on numerous parameters. Even, if the underlying basic computa-

tional problems are intractable, some part of the signature scheme might be implemented in such a way that an attack is possible. A famous example for such a situation is the discovery of the possibility of an attack against the RSA encryption standard PKCS # 1 (see [2]). In this standard, an insecure padding scheme was used, which compromised the security of the whole scheme, even though the RSA encryption scheme is based on the intractable integer factoring problem. However, if the security of the digital signature scheme can be reduced to the difficulty of a well defined computational problem in mathematics, then, in order to evaluate the security of the digital signature scheme, it is sufficient to study the difficulty of the underlying problem. The difficulty of the underlying mathematical problem can be studied thoroughly and, therefore, the level of security of the signature scheme is easier to estimate.

Such a security reduction also solves another problem. It is possible that a weakness of a digital signature scheme is discovered, for example, by a government agency of some country. That agency may then try to keep this weakness secret and take advantage of it. However, if the weakness of the signature scheme implies that a basic computational problem is no longer intractable, then keeping this weakness secret may be more difficult, since the solution of important scientific problems can be expected to happen at the same time in different places. Hence, reduction proofs make it less likely that a security hole can be abused.

It is an important question what the computational problems are to which the security of digital signature schemes should be reduced in order for the scheme to be considered more secure. Clearly, breaking a digital signature scheme in one of the ways explained below, can be considered to be a computational problem. In this sense, the security of any digital signature scheme can be trivially reduced to the intractability of a computational problem, namely to the problem of breaking itself. However, evaluating the computational difficulty of this problem is very difficult,since it is very complex and has many parameters. This is even more true since breaking a digital signature scheme is a so called *interactive problem*, that is, in that problem several parties are involved: a signer, who knows his secret key, an attacker who does not know that key but wants to generate valid signatures of the signer, and perhaps an honest verifier. In the process of forging signatures the attacker can try to use the help of the signer and the verifier (see Section 3.1.5).

To make the security level of a digital signature scheme easy to evaluate,

it is desirable to reduce its security to easy to specify *non-interactive* computational problems. An example is the factoring problem for RSA-modules: Given an integer n which is the product of two large primes p and q, find those factors p and q. It would be optimal to reduce the security of a digital signature scheme to problems which are of mathematical interest independently of their cryptographic applications. Then, the difficulty of those problems would be studied also outside the crypto community and would therefore be easier to evaluate. However, digital signature schemes whose security can be reduced to such problems seem not to be known. In the known reductions, the computational problems depend to a certain extent on the specific digital signature scheme whose security is reduced to them. This is also true in our context and I will discuss this below. In my opinion, the less the computational problems depend on the digital signature schemes the stronger the security proof by reduction is.

3.1.2 Security of the secret key

A minimum requirement for secure digital signature schemes is the security of the secret key. An attacker has access to the public key of the signer. In a secure digital signature scheme, the determination of the secret key from the public key must be infeasible. In the digital signature scheme under review the security of the secret key can be reduced to well studied intractable problems. This will be explained below.

3.1.3 Existential forgery

Suppose that the problem of computing the secret key from the public key is intractable. This does not necessarily mean that the digital signature scheme is secure. It may still be possible that an attacker is able to generate valid signatures without the knowledge of the secret key.

In an *existential forgery* the attacker produces such a signature. In such a forgery, the attacker is not required to have control over the document which is signed. The only requirement is, that the result of an existential forgery a new signature of some document which has been produced without the knowledge of the secret key.

3.1.4 No message attacks

A *no message attack* or a *passive attack* is an existential forgery in which the attacker only knows the public key of the signer and has no access to further information such as valid signatures of other documents. A digital signature scheme is considered to be secure against no message attacks, if the possibility of such an attack implies the ability of solving a computational problem which is considered to be intractable.

3.1.5 Adaptive chosen message attacks

I explain the strongest security notion known for digital signature schemes: the security against existential forgery using an *adaptive chosen message attack*.

In an *adaptive chosen message attack* the adversary knows the public signature key of the signer and obtains valid signatures of a sequence of messages of his choice. The messages in the sequence may depend on signatures of previous messages. The goal of the adversary is an *existential forgery*, i.e. he wants to produce a new signature which has not been generated by the legitimate signer. In particular, the signature is not in the sequence of messages whose signatures the attacker has obtained. But the newly signed message is not necessarily a message of the attackers choice.

One practical application of this notion is as follows. Suppose that a signature scheme is used in a challenge response identification, for example in the ESIGN identification. Then the verifier generates challenges which the prover is supposed to sign, thereby proving his identity. Those challenges can be generated as a sequence of adaptive chosen messages. If an adaptive chosen message attack makes existential forgery possible, then the verifier is able to forge valid signatures without knowing the secret key. The use of signature schemes in challenge response identification is quite common.

I explain a method for proving security against chosen message attacks more precisely. In a chosen message attack the attacker can generate a sequence of pairs (message, signature). A message in that sequence may depend on the previous pairs. The signature generation algorithm is probabilistic. Therefore, the signatures are generated according to some probability distribution. The signature scheme is considered secure against a chosen message attack if it is secure against no message attacks and if without using the secret key it is possible to generate a sequence which is algorithmically indis-

tinguishable (see [1]) from the sequence which is generated using the signature algorithm. The idea of this concept is the following. If an existential forgery is possible using an adaptively chosen sequence of pairs (message, signature), then an existential forgery is possible using the algorithmically indistinguishable simulation of such a sequence. This latter existential forgery is a no message attack since the signing algorithm is not used. However, the digital signature scheme is known to be secure against no message attacks. Therefore, an adaptive chosen message attack is impossible.

It is common belief that security against adaptive chosen message attacks is the strongest possible security notion for digital signature schemes. In other words, no attack against a digital signature scheme is known which cannot be modeled as an adaptive chosen message attack. The role of this security notion is somewhat similar to the role of the model of a Turing machine in the theory of computation. No computing device is known which cannot be modeled as a Turing machine. However, no proof is known that no stronger computing model exists. Likewise, no proof is known that the security against adaptive chosen message attacks is the strongest possible security notion.

In my opinion, if a signature scheme is proven secure against adaptive chosen message attacks, then it can be considered secure in the strongest sense. However, there are no such proofs but only reductions (see Section 3.1.1).

3.1.6 Random oracle model

Security proofs for digital signature schemes are difficult since a digital signature scheme consists of many components and their interaction may be complicated. An important ingredient of most signature schemes are cryptographically secure hash functions. The hash functions map very long messages to short strings of fixed length. The security of hash functions is discussed in Section 4.4. There I explain that no provably secure cryptographic hash functions are known.

If the security of a signature scheme is analyzed in the random oracle model (see [12], [4]), then the concrete hash function which is used in the digital signature scheme, is replaced by a so called *random oracle*. A random oracle can be viewed as a black box which contains a random function which maps long strings to short strings of fixed length. Nothing is known about this function, but it can be evaluated by making an explicit query. A typical

proof of security against passive attacks in the random oracle model works as follows. If it is possible to come up with a forged signature for a document using one random oracle then such a forgery is also possible with another random oracle, resulting in another falsified signature (forking lemma, see [12]). The two valid signatures of the same document can then be used to solve an underlying mathematical problem.

Does a security proof in the random oracle imply the security of the real digital signature scheme in which a concrete hash function is used? Such an implication cannot be proved today. However, assuming that the concrete hash function behaves like a random oracle, a security proof in the random oracle model makes the security of the real scheme more plausible. On the other hand, there exist insecure signature schemes that can be proved secure in the random oracle model (see [4]). Those schemes look fairly artificial. Nevertheless, their existence raises the question what security proofs in the random oracle model really prove.

In my opinion, security proofs in the random oracle cannot prove the security of digital signature schemes but they make their security more plausible.

3.2 My-Ellty ECMR

In Section 5.3.2 I show that because of the choice of the hash function all versions of My-Ellty can be attacked by a birthday attack. Nevertheless, I explain the results from the security analysis with respect to possible reductions.

3.2.1 Security of the secret key

Computing the secret My-Ellty key from the public My-Ellty key is equivalent to solving the discrete logarithm problem in the elliptic curve point group specified in the My-Ellty variants.

3.2.2 Security reductions

The self evaluation shows that in the random oracle model, My-Ellty is secure against no message attacks as long as the discrete logarithm problem in the

point group of one of three elliptic curves, which are specified in the My-Ellty variants, are intractable.

This security argument is not convincing since the chosen hash functions cannot be viewed as random oracles. Also, nothing is proved concerning the security of the My-Ellty schemes against adaptive chosen message attacks, the strongest security notion.

Chapter 4

Basic computational problems

In this chapter I describe the basic computational problems which, given current knowledge, have to be solved in order for the digital signature schemes under review to be insecure and I evaluate the difficulty of solving those problems.

4.1 Basics

In this section I explain the terminology which is used in this chapter.

4.1.1 Asymptotic complexity

To estimate the running time and storage requirement of the algorithms that solve the basic problems the function

$$L_x[u, v] = e^{v(\log x)^u (\log\log x)^{1-u}}$$

is used, where x, u, v are positive real numbers. I explain the meaning of this function. We have

$$L_x[0, v] = e^{v(\log x)^0 (\log\log x)^1} = (\log x)^v \tag{4.1}$$

and

$$L_x[1, v] = e^{v(\log x^1 (\log\log x)^0} = e^{v \log x}. \tag{4.2}$$

Let x be a positive integer which is the input for an algorithm. In the context of this evaluation, x is the cardinality of the finite field over which an elliptic curve is considered. The binary length of x is $\lfloor \log_2 x \rfloor + 1$.

If an algorithm has running time $L_x[0, v]$, then by (4.1) it is a polynomial time algorithm. Its complexity is bounded by a polynomial in the size of the input. The algorithm is considered efficient, although its real efficiency depends on the degree v of the polynomial.

If the algorithm has running time $L_x[1, v]$, then by (4.2) it is exponential. Its complexity is bounded by an exponential function in the length of the input. The algorithm is considered inefficient.

If the algorithm has running time $L_x[u, v]$ with $0 < u < 1$, then it is *subexponential*. The algorithm is slower than polynomial but faster than exponential. So the function $L_x[u, v]$ can be viewed as a linear interpolation between polynomial time and exponential time.

4.1.2 Practical run times

As usual, the experimental run times of the algorithms are given in *MIPS Years*. One MIPS Year is defined as the amount of computation that can be performed in one year by a single DEC VAX 11/780. Using this terminology, one year of computing on an n-MHz PC is comparable to n MIPS Years. However, this is only a rough estimate of the computing power used, since the computation may have space intensive parts, such as the solution of large linear systems, which cannot be executed on a PC.

4.1.3 Quantum computers

In the early 1980s, Richard Feynman (among others) introduced the idea of a new computing device which is based on the laws of quantum mechanics. Peter Shor [14] was able to prove that on such a quantum computer the integer factoring problem (IFP) and the discrete logarithm problem in finite fields have polynomial time solutions. This means that all cryptosystems under consideration here and, more generally, all public-key cryptosystems which are currently being used in practice, are insecure, if quantum computers become practical. There are first experiments with quantum computers, for example at Los Alamos. However, it is unclear whether quantum computers will ever be practical. For the time being, quantum attacks are not feasible. However, it is necessary to watch the development in the area of quantum computing. Also, it appears to be necessary to develop new digital signature schemes which remain secure even if quantum computers become practical.

4.2 Discrete logarithms on elliptic curves

The security of ECDSA and My-Ellty ECMR depends on the intractability of the discrete logarithm problem on elliptic curves over finite fields (ECDL) which I discuss in this section.

4.2.1 The problem

Cryptography with elliptic curves over finite fields (used in ECDSA and My-Ellty) uses the following setting. F is a finite field of cardinality q. E is an elliptic curve over F. G is a point on E of prime order l. The *domain parameters* F, E, G, and l are publicly known.

The elliptic curve discrete logarithm problem (ECDL) is the following: Given a point Y in the subgroup generated by G, find an integer $v \in \{0, \ldots, l-1\}$ such that $Y = vG$.

By a theorem of Hasse, the order of the group of points on E over F is $q + 1 - t$ with $|t| \leq 2\sqrt{q}$. The number t is called the *trace* of the curve (see [10]).

4.2.2 Square root attacks

The only known general purpose algorithms for ECDL are *generic DL-algorithms*. They work in any cyclic group as long as it is known, how the group elements are multiplied, inverted and how the equality of group elements can be decided.

If the group order including its prime factorization is known, then the Pohlig-Hellman algorithm reduces the DL problem in the full group in polynomial time to discrete logarithm problems in groups whose orders are the prime divisors of the group order (see [3]). Since the order of the elliptic curve point group generated by P is a prime number, the Pohlig-Hellman algorithm gives no advantage.

The fastest generic discrete logarithm algorithm in a cyclic group of prime order is the parallel Pollard ρ-algorithm (see [17], [5]). Its running time is proportional to $\sqrt{l}/r = L_l[1, 1/2]/r$ where l is the group order and r is the number of processors used. Using this algorithm it was possible to compute discrete logarithms in an elliptic curve point group over a finite prime field where the group order is a 97-bit prime using $2 * 10^{14}$ group operations (see [5]). Also, it was possible to compute the discrete logarithm in an elliptic

curve point group with a 108-bit order over a finite field of characteristic 2 using $2.3 * 10^{15}$ group operations (see [5]).

Using those data points and the complexity of the parallel Pollard ρ-algorithm, Lenstra and Verheul [11] estimate that for the next 20 years ECDL is intractable in an elliptic curve point group of prime order $l > 2^{161}$. If algorithmic progress is taken into account then $l > 2^{188}$ is recommended.

4.2.3 ECDL algorithms for special curves

There are a number of algorithms which solve ECDL for specific classes of curves.

Frey-Rück attack From [7] we obtain the following: Let

$$k = \min\{i \in \mathbb{Z}_{>0} : q^i \equiv 1 \bmod l\}. \tag{4.3}$$

where l, q are as in Section 4.2.1. In other words, k is the order of q in the group $\mathbb{F}_l^*$. Then there is a polynomial time reduction of the discrete logarithm problem on the elliptic curve to the discrete logarithm problem in the multiplicative group of the finite field $\mathbb{F}_{q^k}$. For fixed k the discrete logarithm problem in $\mathbb{F}_{q^k}$ can be solved in time $L_{q^k}[1/3, c]$ for some constant c (see [13]). For small k this subexponential attack is much faster than all known general purpose ECDL-algorithms (see Section 4.2.2). To prevent the Frey-Rück attack, it is necessary to choose k at least $\lceil 2000/\log_2(q) \rceil$. This condition is based on the assumption that the discrete logarithm problem in a finite field, whose cardinality is a 2000-bit number, is intractable. For a 160-bit prime p this means that $k \geq 13$. I remark that the German Information Security Agency requires $k \geq 10^4$ since some researchers feal that more efficient attacks are possible. But no such attacks are known.

Anomalous curve attack If the trace of the curve E is 1, then ECDL can be reduced to a discrete logarithm problem in the additive group $\mathbb{Z}/q\mathbb{Z}$ (see [15]) which can be solved by the extended euclidean algorithm in polynomial time. Therefore, curves of trace 1 must be avoided in cryptographic applications.

Weil descent attacks If $q = 2^{nm}$ with small n such as $n = 4$ then ECDL can be reduced to a DL problem on a hyperelliptic curve which by an algorithm of Gaudry can be solved faster then the general ECDL problem (see [8]). Therefore, those field sizes must be avoided.

4.2.4 Quantum attacks

Using the methods from [14] it is possible to show that ECDL can be solved in polynomial time on a quantum computer. It is not yet clear whether quantum computers become ever practical. Currently, quantum attacks do not threaten ECDL.

4.3 Random and pseudorandom number generation

The key and signature generation in the My-Ellty schemes require the generation of random numbers, specifically random primes. They are in general generated as sequences of random bits.

Such a sequence is generated as follows. A random bit generator (RBG) is used to generate a short sequence of true random bits. Since a RBG is too inefficient, the short true random sequence is expanded by a pseudorandom bit generator (PRBG) into a sequence of the necessary length.

The RBGs used by the schemes under review are not described in the submission. I can therefore not discuss their security here. However, in real applications it is necessary that a cryptographically secure RBG is used.

A PRBG receives as input a random bit sequence and outputs a longer pseudorandom bit sequence. A PRBG is cryptographically secure if an attacker is not able to distinguish its output from a true random sequence in polynomial time.

No provably secure PRBG is known. Several PRBGs are known whose security can be reduced to the intractability of certain number theoretic problems such as the discrete logarithm problem in finite fields (see [3]). However, those PRBGs are not sufficiently efficient.

The PRBG used in practice survive a broad class of statistical tests specified, for example, in [6].

4.4 Hash functions

In signature schemes, hash functions are used to map long documents to short bit strings of a fixed length, which are actually signed. A *collision* of a hash function is a pair of different documents which are mapped to the same hash value. A hash function is called *collision resistant* if finding a collision of that hash function is intractable.

In signature schemes, which sign hash values, the used hash functions must be collision resistant. Otherwise, if a collision (d, d') is found then the two documents d and d' have the same signature. If an attacker is able to obtain a valid signature of d, then he has also a signature for d'. For example, if the signer signs d in a challenge-response authentication, then he has also signed the other document d', possibly without knowing it. Collision resistant hash function are one way functions. This means, that computing an inverse image for a given image is intractable. Therefore, in many cases, the use of collision resistant hash functions prevents existential forgeries since even if it is possible to generate a valid signature for a hash value it is impossible to find a document with that hash value.

Using the birthday paradox (see [3]) a collision for a hash function whose image has n elements can be found with probability $> 1/2$ by computing approximately $\sqrt{n}$ hash values. Therefore, the image of the hash function should at least contain 2^{160} elements.

No hash function is known for which the birthday attack is provably the only possible attack. In the past, hash functions such as MD4 have been shown not to be collision resistant. Today, the hash functions SHA-1 [16] and RipeMD-160 [9] are used in practice. Given current knowledge, they are collision resistant.

The birthday attack can be prevented if a *keyed hash function* is used. This is a function which maps a bit string and a key from a predefined key space to a hash value of fixed length. In the signature process, a random key is generated. The signature algorithm signs the hash value of a document that is generated by the hash function which is parameterized by the chosen key. The key is part of the signature. It is also used in the verification process. In order for digital signature algorithm to be secure, the keyed hash function must be a *universal one-way hash function* (UOWF). This means that given a hash value and a key it is intractable to find a document such that the value of the hash function parameterized by the given key is the given hash value. No provably universal one-way hash function is known. However, there are

constructions that use compression functions such as SHA-1 (see [16]) and are assumed to have the universal one-way property.

Chapter 5

Basic cryptographic problems

In this chapter I discuss the hardness of the non-interactive computational problems which are the basis of the security of the My-Ellty schemes.

5.1 My-Ellty ECMR OEF-h

This version of My-Ellty works on an elliptic curve over a so called *optimal extension field*. Such a field is an extension field of a prime field with 32-bit characteristic. This choice is supposed to hasten the arithmetic on the curve. The arithmetic in the ground field uses built-in 32-bit operations which is fast.

5.1.1 My-Ellty ECMR OEF-h-DL

The prime number $p = 2^{32} - 185$, a specific elliptic curve $E(p, a, b)$ over $\mathbb{F}_{p^5}$ and a point G on that elliptic curve of prime order l are given. The prime l is explicitely presented. I have verified those data.

The secret My-Ellty ECMR-OEF-h key can be computed from the public My-Ellty ECMR-OEF-h key, if the discrete logarithm problem in the subgroup generated by G can be solved.

The choices of the parameters prevent the use of the Frey-Rück attack and the anomalous curve attack. The Frey-Rück attack is prevented since k from (4.3) is a 160-bit number and I have explained in Section 4.2.3 that $k \geq 13$ is sufficient. The anomalous curve attack is prevented since the trace of that curve is different from 1.

The Weil descent attack gives no advantage over fields of characteristic p^5.

Since l is a 160-bit prime square root attacks are infeasible for the next 20 years (see [11]). Therefore, given the current algorithmic knowledge, ECDL in the subgroup generated by G will remain intractable for the next 20 years.

5.1.2 Hash function

The hash function h used in My-Ellty ECMR OEF-h maps all documents to 80-bit hash values. Therefore, a birthday attack is feasible (see Section 4.4) that is, two different documents d and d' can be found which map to the same hash value. A birthday attack can be mounted on My-Ellty ECMR OEF-h as follows. Select an 80-bit string s. Find two different bit strings x and x' such that $h(s||x) = h(s||x')$. This is possible using the birthday attack since $h' : \{0,1\}^* \to \{0,1\}^{80}$, $x \mapsto h(s||x)$ is a hash function with 80-bit output. By the construction of My-Ellty ECMR OEF-h the signatures of $d = s||x$ and $d' = s||x'$ are the same.

5.1.3 Random number generator

No specific random number generator is specified. However, it is recommended that the random number generator satisfies the requirements of FIPS-140 (see [6]).

5.2 My-Ellty ECMR 160-h

5.2.1 My-Ellty ECMR 160-h-DL

The prime number $p = 2^{160} - 33689$, an elliptic curve $E(p, a, b)$ over $\mathbb{F}_p$, and a point G on that elliptic curve of prime order l are given. The prime l is explicitely presented. I have verified those data.

The secret My-Ellty ECMR-160-h key can be computed from the public My-Ellty ECMR-160-h key, if the discrete logarithm problem in the subgroup generated by G can be solved.

The choices of the parameters prevent the use of the Frey-Rück attack and the anomalous curve attack. The Frey-Rück attack is prevented since k from (4.3) is a 158-bit number and I have explained in Section 4.2.3 that

$k \geq 13$ is sufficient. The anomalous curve attack is prevented since the trace of that curve is different from 1.

The Weil descent attack is not applicable over prime fields.

Since l is a 160-bit prime square root attacks are infeasible for the next 20 years (see [11]). Therefore, given the current algorithmic knowledge, ECDL in the subgroup generated by G will remain intractable for the next 20 years.

5.2.2 Hash function

The hash function h used in My-Ellty ECMR 160-h maps all documents to 80-bit hash values. Therefore, a birthday attack is feasible (see Section 4.4) that is, two different documents d and d' can be found which map to the same hash value. A birthday attack can be mounted on My-Ellty ECMR 160-h as follows. Select an 80-bit string s. Find two different bit strings x and x' such that $h(s||x) = h(s||x')$. This is possible with the birthday attack since $h' : \{0,1\}^* \to \{0,1\}^{80}$, $x \mapsto h(s||x)$ is a hash function with 80-bit output. By the construction of My-Ellty ECMR 160-h the signatures of $d = s||x$ and $d' = s||x'$ are the same.

5.2.3 Random number generator

No specific random number generator is specified. However, it is recommended that the random number generator satisfies the requirements of FIPS-140 (see [6]).

5.3 My-Ellty ECMR 192-h

5.3.1 My-Ellty ECMR 192-h-DL

The prime number $p = 2^{192} - 34757$, an elliptic curve $E(p, a, b)$ over $\mathbb{F}_p$, and a point G on that elliptic curve of prime order l are given. The prime number l is explicitely presented. I have verified those data.

The secret My-Ellty ECMR-192-h key can be computed from the public My-Ellty ECMR-192-h key, if the discrete logarithm problem in the subgroup generated by G can be solved.

The choices of the parameters prevent the use of the Frey-Rück attack and the anomalous curve attack. The Frey-Rück attack is prevented since k

from (4.3) is a 192-bit number. I have verified this number. The anomalous curve attack is prevented since the trace of that curve is different from 1.

The Weil descent attack is not applicable over prime fields. Since l is a 192 bit prime square root attacks are infeasible for the next 20 years (see [11]). Therefore, given the current algorithmic knowledge, ECDL in the subgroup generated by G will remain intractable for the next 20 years.

5.3.2 Hash function

The hash function h used in My-Ellty ECMR 192-h maps all documents to 96-bit hash values. Therefore, a birthday attack is feasible (see Section 4.4) that is, two different documents d and d' can be found which map to the same hash value. A birthday attack can be mounted on My-Ellty ECMR 192-h as follows. Select a 96-bit string s. Find two different bit strings x and x' such that $h(s||x) = h(s||x')$. This is possible with the birthday attack since $h' : \{0,1\}^* \to \{0,1\}^{96}$, $x \mapsto h(s||x)$ is a hash function with 96-bit output. By the construction of My-Ellty ECMR 192-h the signatures of $d = s||x$ and $d' = s||x'$ are the same.

5.3.3 Random number generator

No specific random number generator is specified. However, it is recommended that the random number generator satisfies the requirements of FIPS-140 (see [6]).

Chapter 6

Final evaluation

In the random oracle model, the security of the My-Ellty schemes against no message attacks can be reduced to the discrete logarithm problem in the point group of the elliptic curves specified in My-Ellty. The random oracle model ignores the concrete hash function which in the case of My-Ellty appears to be insecure (see Section 5.2). Replacing the hash function in My-Ellty by a more secure one may result in losing the partial message recovery property of My-Ellty. Also, no reductions of the security against adaptive chosen message attacks are are presented.

The security of the My-Ellty schemes is based on ECDL, the discrete logarithm problem the point group of an elliptic curve over a finite field. In the My-Ellty schemes, the curves are precisely specified. ECDL with those curves appears to be intractable for the next 20 years. However, the My-Ellty curves are not in the standards. Therefore, they have not received as much attention by the cryptanalytic community as the standardized curves.

The hash function used in the My-Ellty schemes admits a birthday attack (see Section 5.3.2). Therefore, the My-Ellty schemes appear to be insecure in their present form.

Not much is said in the My-Ellty submission about the generation of random numbers or pseudorandom numbers. The submissions refer to the corresponding standards. Therefore, with respect to the random number generation, the proposed schemes are of comparable security.

In the present form, I cannot recommend the use of the My-Ellty schemes since they admit a birthday attack.

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